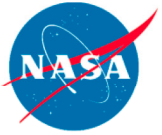*National Aeronautics and Space Administration*

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

SAR POLARIMETRY

Jakob J. van Zyl

Jet Propulsion Laboratory

California Institute of Technology

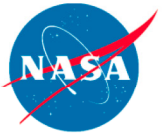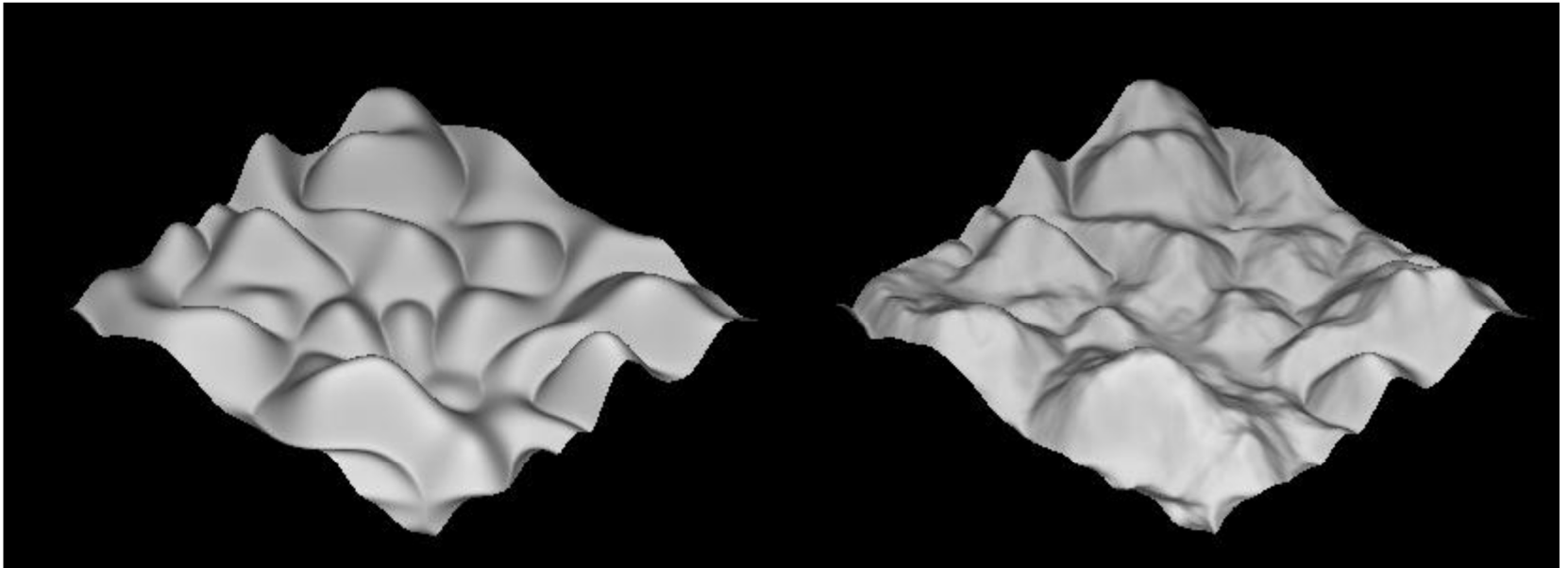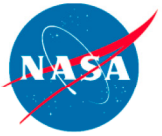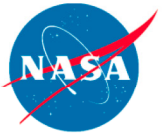4800 Oak Grove Drive

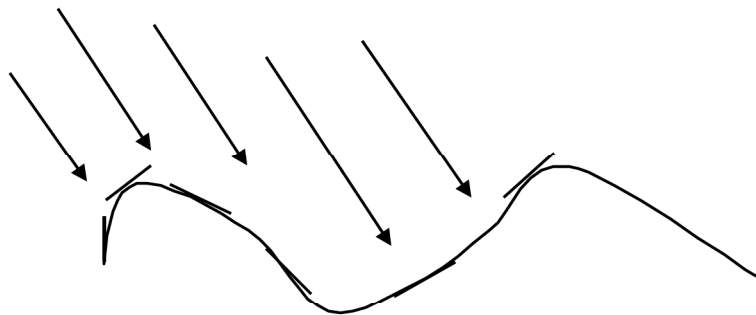Pasadena, CA 91109

Section 1

RADAR SCATTERING

Surface Characteristics
Geometrical Properties
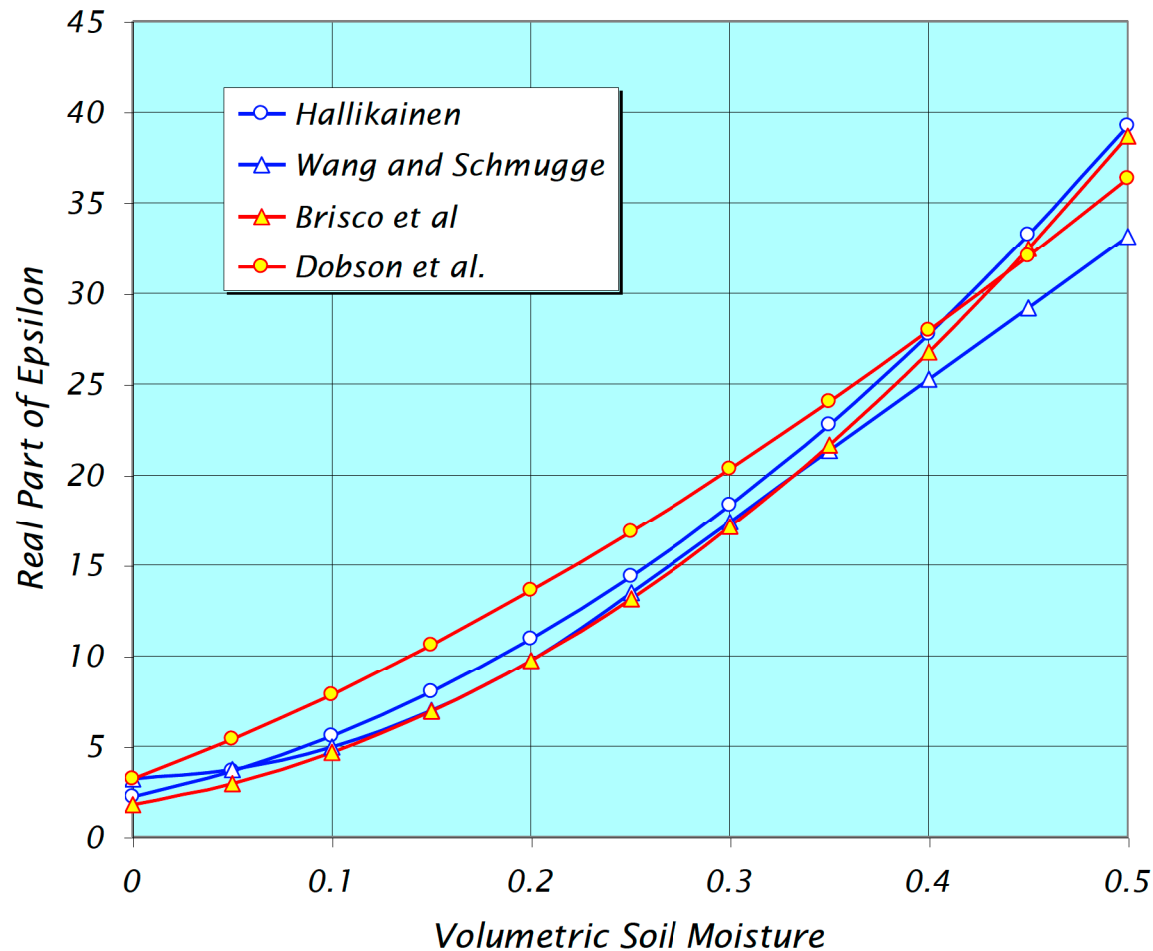
Rough Surface Scattering

As surface roughness increases, the angular
spread of a scattered wave increases.
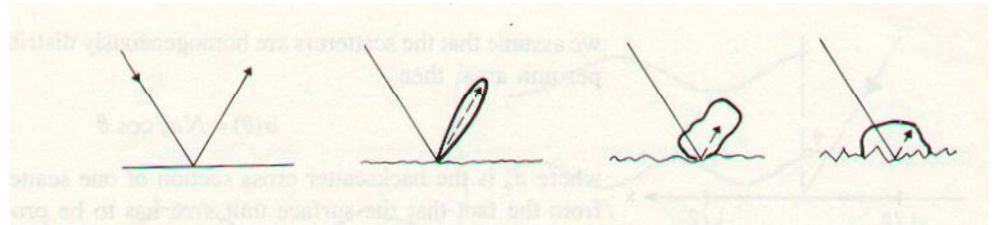
Scattering from facets:

Geometric Optics solution

$2k\sin\theta$
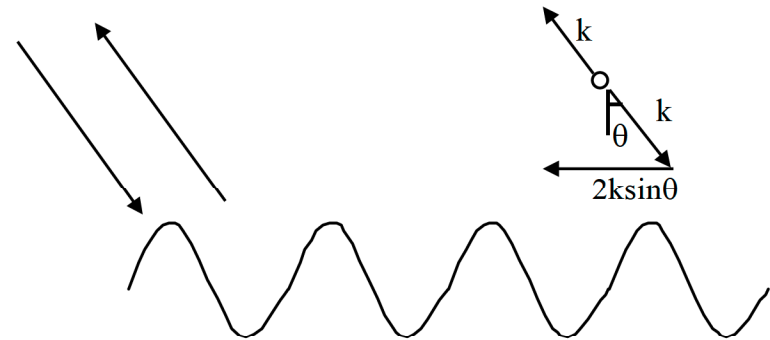
Resonant scattering:
Bragg scattering

- References: P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough
  Surfaces*, Artech House, 1987.
- C. Elachi, *Spaceborne Radar Remote Sensing: Applications and Techniques,* IEEE Press, New York, 1988

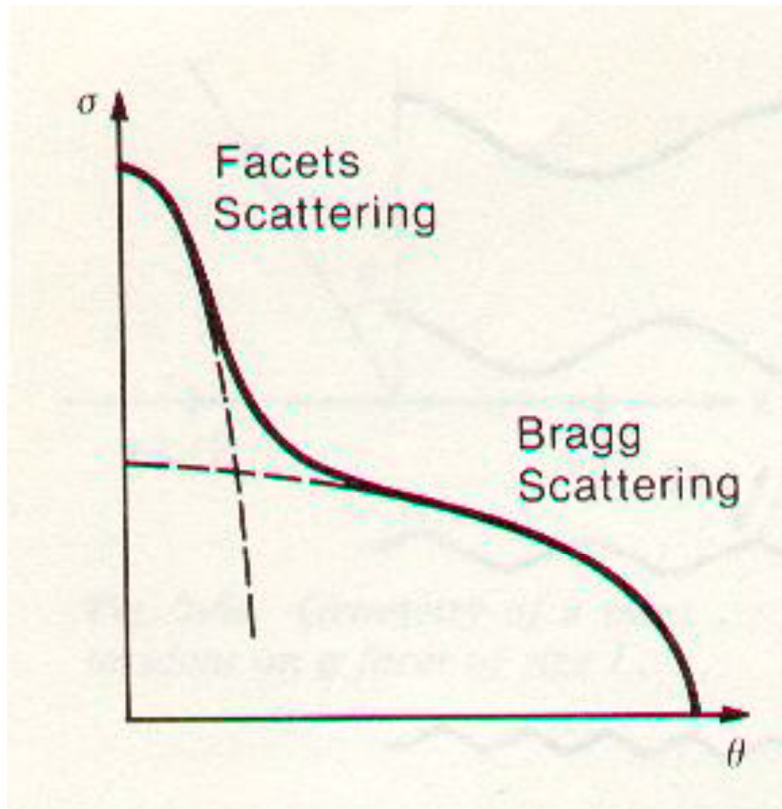# Geometrical Optics And Small Perturbation Method Solutions

C. Elachi, *Spaceborne Radar Remote Sensing: Applications and Techniques,* IEEE Press, New York, 1988

- Geometrical Optics Solution

$$\sigma_{GO} = \frac{|\rho(0)|^2 e^{-\tan^2\theta/s^2}}{s^2 \cos^4\theta}$$

$\rho(0)$: normal reflection coefficient

s^2 : two dimensional slope variance $(= s_x^2 + s_y^2)$

- Bragg Scattering Solution (Small Perturbation Method)

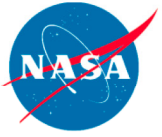$$\sigma_{Bragg} = 8k^4 h^2 \cos^4\theta |\alpha_{tr}|^2 W_n(2k\sin\theta)$$

$$k = \frac{2\pi}{\lambda} \quad h = \text{rms height}$$

$$\alpha_{hh} = \frac{1-\varepsilon}{\left[\cos\theta + \sqrt{\varepsilon - \sin^2\theta}\right]^2}$$

$$\alpha_{vv} = \frac{(\varepsilon-1)[\sin^2\theta - \varepsilon(1+\sin^2\theta)]}{\left[\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}\right]^2}$$

Normalized power spectrum: $\int W_n(\kappa)\kappa d\kappa = 1$

Reference: F. T. Ulaby, R. K. Moore, and A. K. Fung, Microwave Remote Sensing, Vol. II, Artech House, 1982

Integral Equation Method

- The integral equation method is a single model that includes both facet and Bragg components. The expressions are

$$\sigma_{xy} = \frac{k^2}{2} \exp\left(-2k^2 h^2 \cos^2\theta\right) \sum_{n=1}^{\infty} h^{2n} \left|I_{xy}^n\right|^2 \frac{W^n\left(-2k\sin\theta,0\right)}{n!}$$

$$I_{xy}^n = \left(2k\cos\theta\right)^n f_{xy} \exp\left(-k^2 h^2 \cos^2\theta\right) + \frac{k^n \cos^n\theta\left[F_{xy}\left(-k\sin\theta,0\right) + F_{xy}\left(k\sin\theta,0\right)\right]}{2}$$

$$W^n\left(k\right) = \frac{2}{\pi} \int_0^{\infty} r\rho^n\left(r\right) J_o\left(kr\right) dr$$
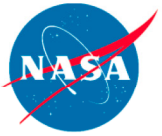
$$f_{hh} = \frac{-2R_h}{\cos\theta}; \quad f_{vv} = \frac{2R_v}{\cos\theta}; \quad f_{hv} = f_{vh} = 0$$

$$F_{hh}\left(-k\sin\theta,0\right) + F_{hh}\left(k\sin\theta,0\right) = \frac{-2\sin^2\theta\left(1+R_h\right)^2}{\cos\theta}\left[\left(1-\frac{1}{\mu}\right) + \frac{\mu\in -\sin^2\theta - \mu\cos^2\theta}{\mu^2\cos^2\theta}\right]$$
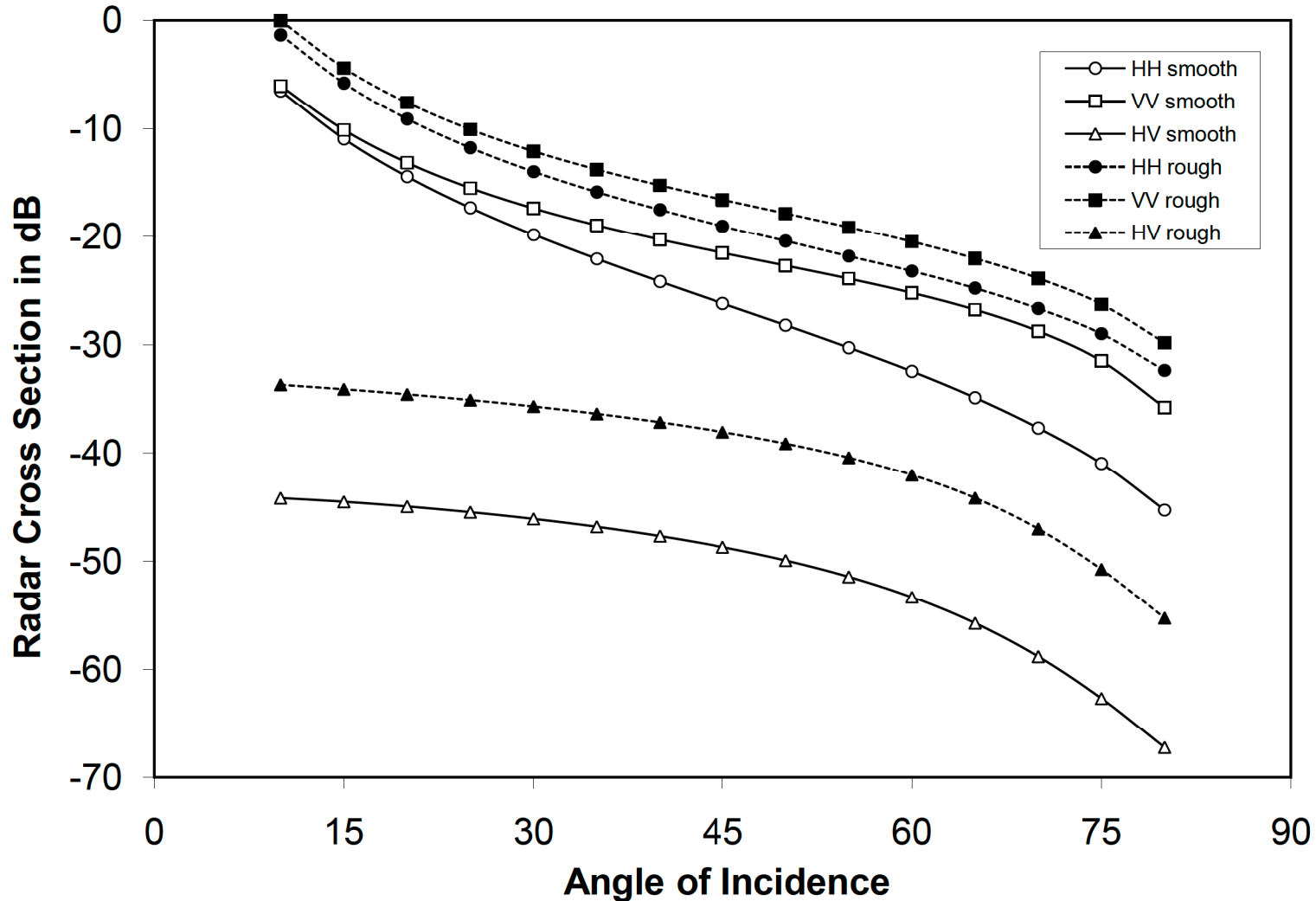
$$F_{vv}\left(-k\sin\theta,0\right) + F_{vv}\left(k\sin\theta,0\right) = \frac{2\sin^2\theta\left(1+R_v\right)^2}{\cos\theta}\left[\left(1-\frac{1}{\in}\right) + \frac{\mu\in -\sin^2\theta - \in\cos^2\theta}{\in^2\cos^2\theta}\right]$$
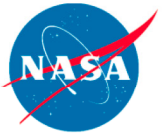
$$F_{hv}\left(-k\sin\theta,0\right) + F_{hv}\left(k\sin\theta,0\right) = 0$$

*Reference: Fung, A. K., Z. Li, K. S. Chen. Backscattering from a Randomly Rough Dielectric Surface, IEEE Trans. Geoscience and Remote Sensing, **30**, 356-369, 1992.*
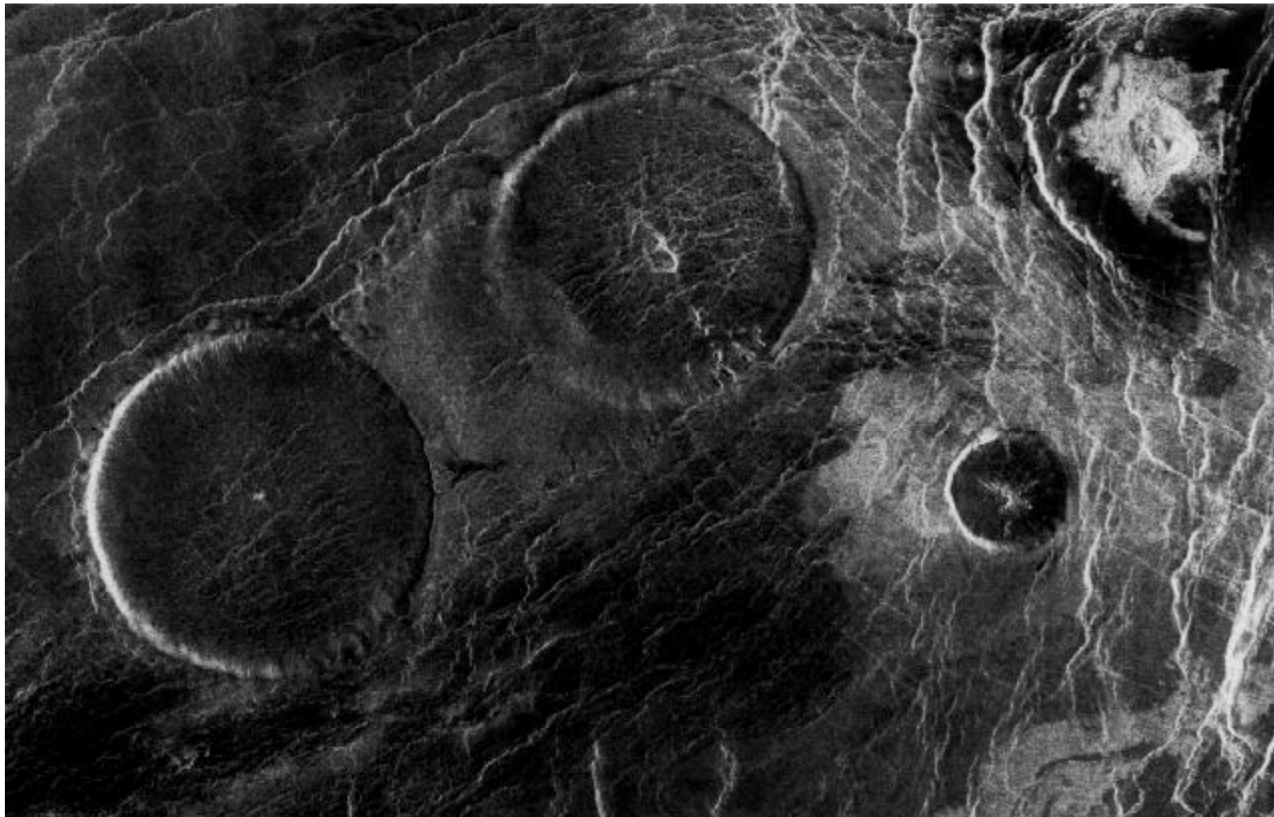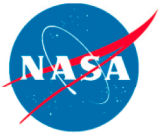
Rough Surface Scattering: Effect of Roughness

Magellan Image of Pancake Domes on Venus

Dickinson Impact Crater on Venus (Magellan)

Lakes on Titan (Cassini Radar)

Longitudinal Dunes on Titan (Cassini Radar)

JvZ-12

Frequency Comparison:
Death Valley, California

C-Band (5.7 cm)

L-Band (24 cm)

JvZ-13

Surface Roughness

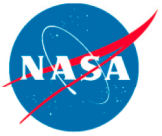# Rough Surface Scattering:
Effect of Dielectric Constant

Surface Dielectric Constant

PISAS: Dielectric Constant (ALGORITHM: DUBOIS et al.)

Surface Dielectric Constant

L-Band

P-Band

0.0

30.0

PISAS: Dielectric Constant (ALGORITHM: DUBOIS et al.)

JvZ-17

Vegetation Scattering

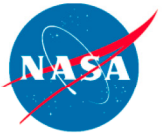- Modeling the scattering from vegetated surfaces is considerably more complicated.

- Radiative Transfer models use radiative transfer algorithms to build the scattering from a layer of scatterers

- Discrete scatterer models approximate the scattering by a collection of randomly oriented scatterers such as cylinders and/or disks and needles over a ground surface
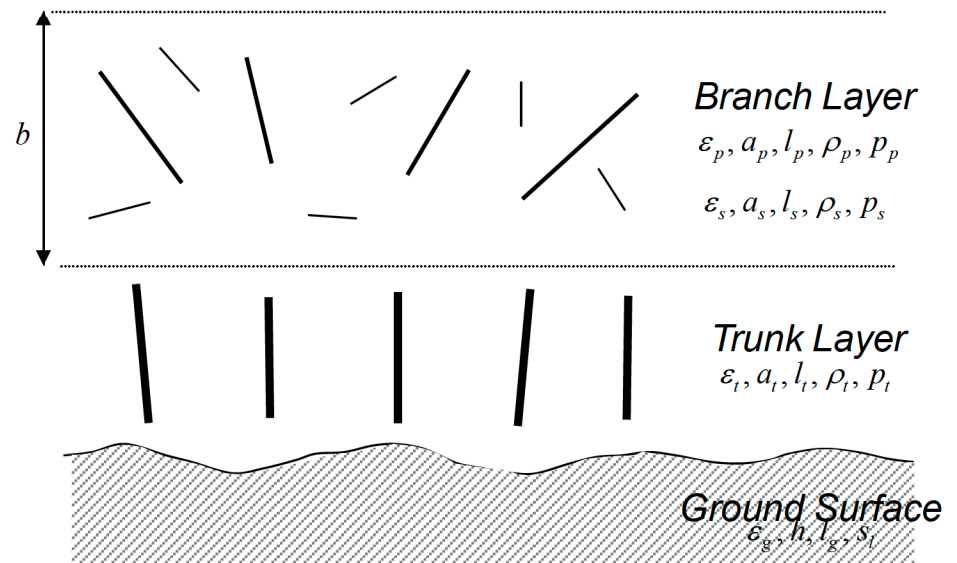
Branch Layer
$\varepsilon_p, a_p, l_p, \rho_p, p_p$

$\varepsilon_s, a_s, l_s, \rho_s, p_s$

Trunk Layer
$\varepsilon_t, a_t, l_t, \rho_t, p_t$

Ground Surface
$\varepsilon_g, h, l_g, s_l$

b

Vegetation Scattering:
Discrete Scatterer Models

- The bistatic scattering functions are calculated for each element
- The scattering contributions from different scatterers are added incoherently
- The scatterered energy is averaged over the statistical distributions of the scatterers
- The ground surface contribution is attenuated by the average extinction coefficient of the layer
- All calculations are performed for all polarization combinations
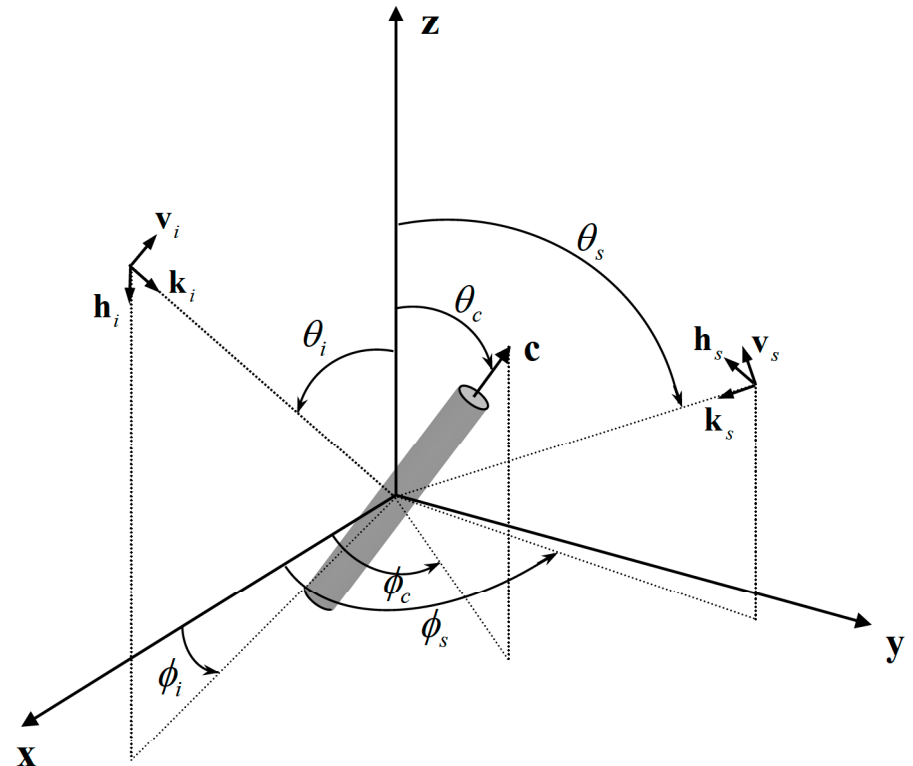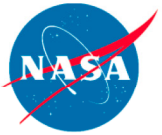
Vegetation Scattering:
Discrete Scattering Models

- In addition to direct scattering from the ground surface (attenuated by the vegetation layer) and the vegetation layer itself, double reflections involving the ground and vegetation elements are also calculated.

- The signal is attenuated by the vegetation layer

- The scattered energy is averaged over all scatterer orientations

Example Results

Slope of Sigma Zero vs Soil Moisture

Frequency Comparison: Vegetated Area
Landes Forest, France

C-Band (5.7 cm)　　**L-Band (24 cm)**　　**P-Band (68 cm)**
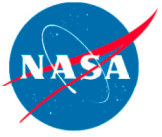
JvZ-24

Vegetation Scattering
L-band Polarization Comparison

HH

VV
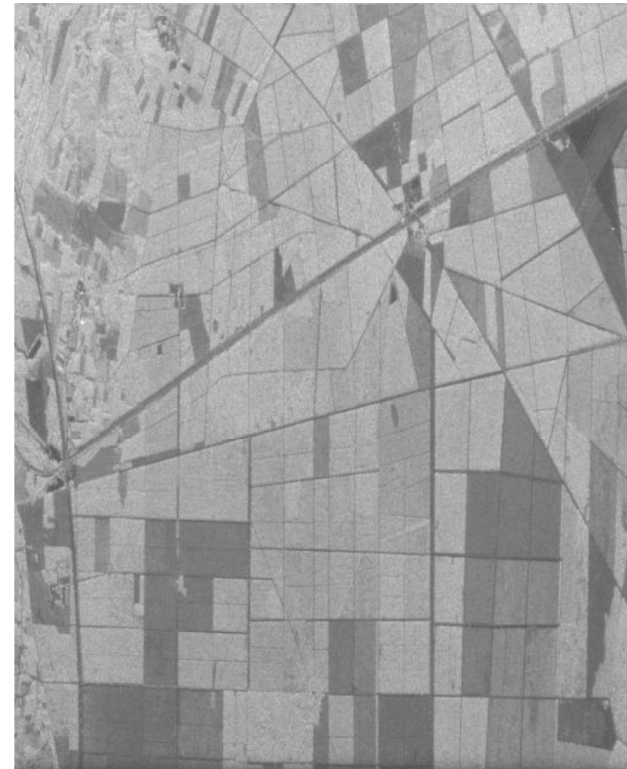
HV
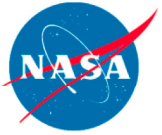
Section 2

POLARIMETRIC RADAR

Principles Of Polarimetry:
Field Descriptions

POLARIZATION VECTOR   **POLARIZATION ELLIPSE**   **STOKES VECTOR**

$$\mathbf{p} = \begin{pmatrix} a_h \\ a_v \end{pmatrix}$$

$$\psi, \chi, a = |a_h|^2 + |a_v|^2$$

$$S = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}$$
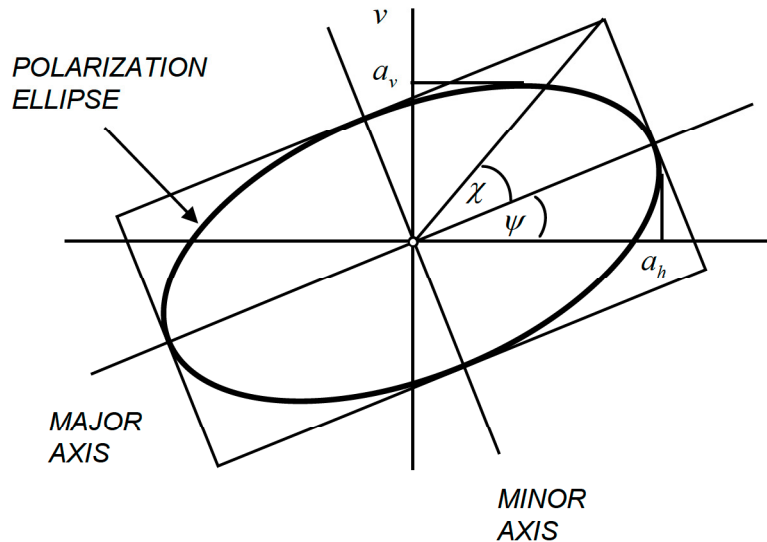
$$S_0 = |a_h|^2 + |a_v|^2$$

$$S_1 = |a_h|^2 - |a_v|^2 = S_0 \cos(2\chi)\cos(2\psi)$$

$$S_2 = 2\Re(a_h a_v^*) = S_0 \cos(2\chi)\sin(2\psi)$$

$$S_3 = 2\Im(a_h a_v^*) = S_0 \sin(2\chi)$$

POLARIZATION
ELLIPSE

MAJOR
AXIS

MINOR
AXIS
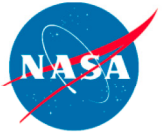
Wave Polarizations:
Geometrical Representations

POLARIZATION
ELLIPSE

v

a_v

χ

ψ

a_h

h

MAJOR
AXIS

MINOR
AXIS

POLARIZATION ELLIPSE

*LEFT-HAND
CIRCULAR*
$\chi = 45^0$

S_3

*LINEAR
POLARIZATION
(VERTICAL)*
$\psi = 180^0 ; \chi = 0^0$

S_0

2χ

2ψ

S_2

S_1

*LINEAR
POLARIZATION
(HORIZONTAL)*
$\psi = 0^0 ; \chi = 0^0$

*RIGHT-HAND
CIRCULAR*
$\chi = -45^0$
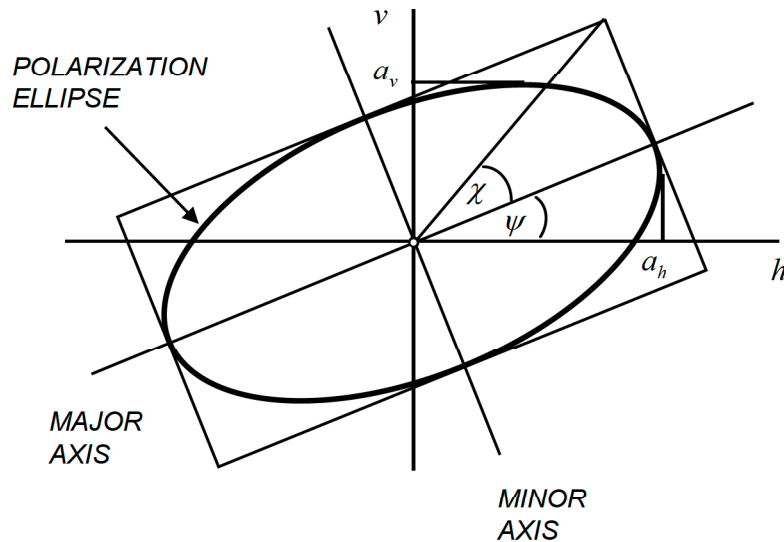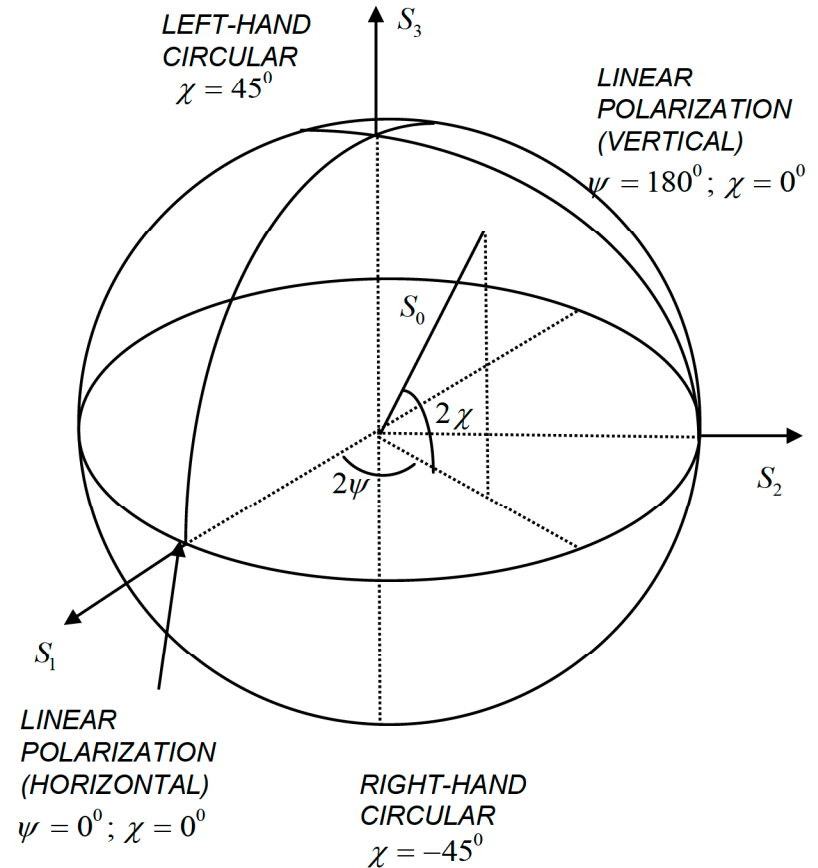
POINCARE SPHERE
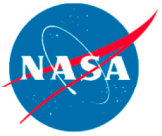
Example Polarizations

LINEAR HORIZONTAL	LINEAR VERTICAL	LEFT-HAND CIRCULAR
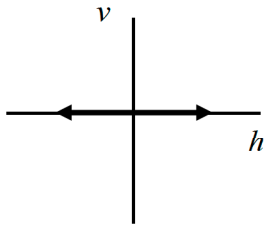
$$\mathbf{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\psi = 0^0; \chi = 0^0$$

$$\mathbf{S} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$
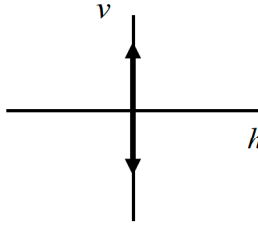
$$\mathbf{p} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi = 90^0; \chi = 0^0$$

$$\mathbf{S} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$
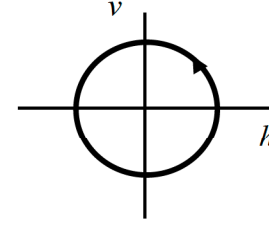
$$\mathbf{p} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\psi = 0^0; \chi = 45^0$$

$$\mathbf{S} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
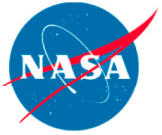
Definition Of Ellipse Orientation Angles

WARNING

Sometimes the polarization ellipse orientation angle is defined with respect to the vertical direction. In that case, linear horizontal polarization has an ellipse orientation angle of +90 degrees or -90 degrees, and linear vertical polarization is characterized by an ellipse orientation angle of 0 degrees. *Both conventions are used in this viewgraph package.*
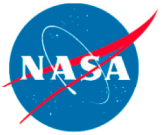
Scatterer As Polarization Transformer

- Transverse electromagnetic waves are characterized mathematically as 2-dimensional complex vectors. When a scatterer is illuminated by an electromagnetic wave, electrical currents are generated inside the scatterer. These currents give rise to the scattered waves that are reradiated.

SCATTERED WAVES

INCIDENT WAVE
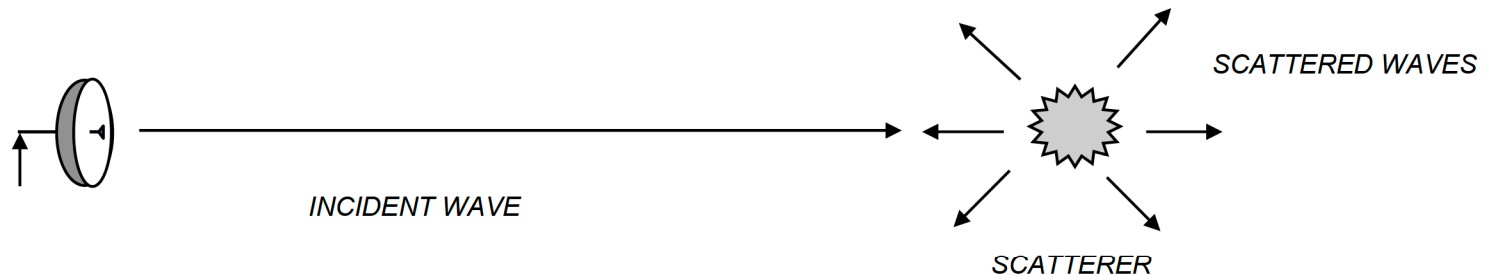
SCATTERER

- Mathematically, the scatterer can be characterized by a 2x2 complex scattering matrix that describes how the scatterer transforms the incident vector into the scattered vector.

- The elements of the scattering matrix are functions of frequency and the scattering and illuminating geometries.
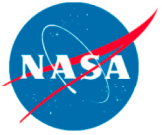
Scattering Matrix

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix}^{sc} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} E_h \\ E_v \end{pmatrix}^{inc}$$

- Far-field response from scatterer is fully characterized by four complex numbers
- Scattering matrix is also known as Sinclair matrix or Jones matrix
- Must measure a scattering matrix for every frequency and all incidence angles
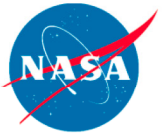
Coordinate Systems

- All matrices and vectors shown in this package are measured using the *backscatter alignment* coordinate system. This system is preferred when calculating radar-cross sections, and is used when measuring them:
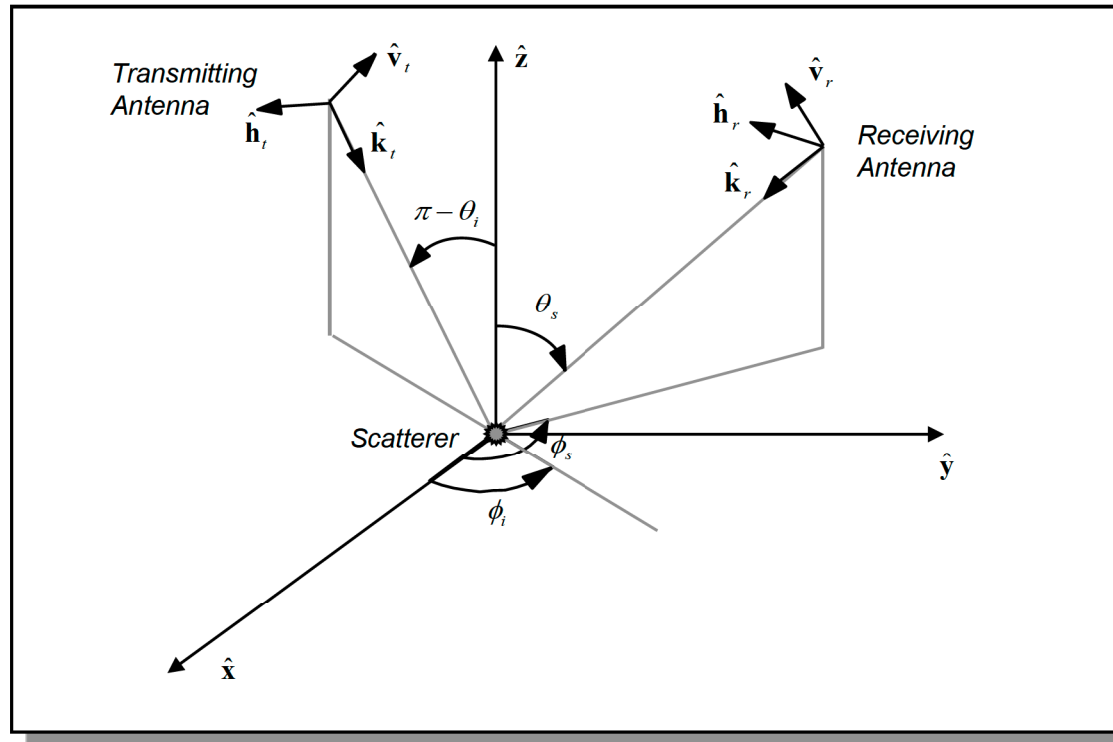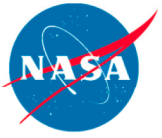
Mathematical Characterization Of Scatterers: Scattering Matrix

- The radiated and scattered electric fields are related through the complex 2x2 scattering matrix:

$$\mathbf{E}^{sc} = [\mathbf{S}]\mathbf{p}^{rad}$$

- The (complex) voltage measured at the antenna terminals is given by the scalar product of the receiving antenna polarization vector and the received wave electric field:

$$V = \mathbf{p}^{rec} \cdot [\mathbf{S}]\mathbf{p}^{rad}$$

- The measured power is the magnitude of the (complex) voltage squared:

$$P = VV^* = \left| \mathbf{p}^{rec} \cdot [\mathbf{S}]\mathbf{p}^{rad} \right|^2$$

NOTE: *Radar cross-section is proportional to power*
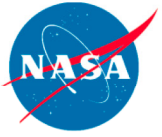
Mathematical Characterization Of Scatterers: Covariance Matrix

- We can rewrite the expression for the voltage as follows:

$$V = \mathbf{p}^{rec} \cdot [\mathbf{S}] \mathbf{p}^{rad}$$

$$= p_h^{rec} p_h^{rad} S_{hh} + p_h^{rec} p_v^{rad} S_{hv} + p_v^{rec} p_h^{rad} S_{vh} + p_v^{rec} p_v^{rad} S_{vv}$$

$$= \begin{pmatrix} p_h^{rec} p_h^{rad} & p_h^{rec} p_v^{rad} & p_v^{rec} p_h^{rad} & p_v^{rec} p_v^{rad} \end{pmatrix} \begin{pmatrix} S_{hh} \\ S_{hv} \\ S_{vh} \\ S_{vv} \end{pmatrix}$$

$$= \tilde{\mathbf{A}}\mathbf{T}$$

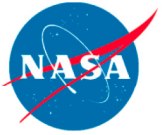- The first vector contains only antenna parameters, while the second contains only scattering matrix elements. Using this expression in the power expression, one finds

$$P = VV^* = \left(\tilde{\mathbf{A}}\mathbf{T}\right)\left(\tilde{\mathbf{T}}\mathbf{A}\right)^* = \tilde{\mathbf{A}}\mathbf{T}\tilde{\mathbf{T}}^*\mathbf{A}^* = \mathbf{A}\cdot[\mathbf{C}]\mathbf{A}^*; \quad [\mathbf{C}] = \mathbf{T}\tilde{\mathbf{T}}^*$$

- The matrix [C] is known as the *covariance matrix* of the scatterer

Pauli Basis and Coherency Matrix

- On the previous slide we expressed the scattering matrix in the so-called h-v basis formed by horizontal and vertical dipoles. One can also express the scattering matrix in vector form in the Pauli basis, given by

$$\vec{p} = \frac{1}{\sqrt{2}} \begin{pmatrix} S_{hh} + S_{vv} \\ S_{hh} - S_{vv} \\ S_{vh} + S_{hv} \\ S_{vh} - S_{hv} \end{pmatrix}$$

- The equivalent form of the covariance matrix in this basis is known as the *coherency matrix*
- The two forms are equivalent and convey the same information about the scatterer. In particular, the eigenvalues are the same.
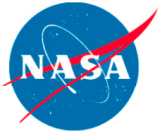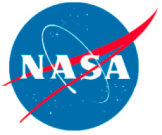
MATHEMATICAL CHARACTERIZATION OF SCATTERERS:
STOKES SCATTERING OPERATOR

- The power expression can also be written in terms of the antenna Stokes vectors. First consider the following form of the power equation:

$$P = \left(\mathbf{p}^{rec} \cdot \mathbf{E}^{sc}\right)\left(\mathbf{p}^{rec} \cdot \mathbf{E}^{sc}\right)^*$$

$$= \left(p_h^{rec} E_h^{sc} + p_v^{rec} E_v^{sc}\right)\left(p_h^{rec} E_h^{sc} + p_v^{rec} E_v^{sc}\right)^*$$

$$= \left(p_h^{rec} p_h^{rec*}\right)\left(E_h^{sc} E_h^{sc*}\right) + \left(p_v^{rec} p_v^{rec*}\right)\left(E_v^{sc} E_v^{sc*}\right) + \left(p_h^{rec} p_v^{rec*}\right)\left(E_h^{sc} E_v^{sc*}\right) + \left(p_v^{rec} p_h^{rec*}\right)\left(E_v^{sc} E_h^{sc*}\right)$$

$$= \begin{pmatrix} p_h^{rec} p_h^{rec*} \\ p_v^{rec} p_v^{rec*} \\ p_h^{rec} p_v^{rec*} \\ p_v^{rec} p_h^{rec*} \end{pmatrix} \cdot \begin{pmatrix} E_h^{sc} E_h^{sc*} \\ E_v^{sc} E_v^{sc*} \\ E_h^{sc} E_v^{sc*} \\ E_v^{sc} E_h^{sc*} \end{pmatrix}$$

$$= \mathbf{g}^{rec} \cdot \mathbf{X}$$

- The vector $\mathbf{x}$ in the expression above is a function of the transmit antenna parameters as well as the scattering matrix elements.

MATHEMATICAL CHARACTERIZATION OF SCATTERERS:
STOKES SCATTERING OPERATOR

- Using the fact that $\mathbf{E}^{sc} = [\mathbf{S}]\mathbf{p}^{rad}$, it can be shown that $\mathbf{X}$ can also be written as
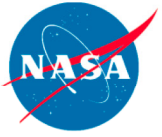
$$\mathbf{X} = [\mathbf{W}]\mathbf{g}^{rad}$$

where

$$[\mathbf{W}] = \begin{pmatrix} S_{hh}S_{hh}^* & S_{hv}S_{hv}^* & S_{hh}S_{hv}^* & S_{hv}S_{hh}^* \\ S_{vh}S_{vh}^* & S_{vv}S_{vv}^* & S_{vh}S_{vv}^* & S_{vv}S_{vh}^* \\ S_{hh}S_{vh}^* & S_{hv}S_{vv}^* & S_{hh}S_{vv}^* & S_{hv}S_{vh}^* \\ S_{vh}S_{hh}^* & S_{vv}S_{hv}^* & S_{vh}S_{hv}^* & S_{vv}S_{hh}^* \end{pmatrix}$$

- This means that the measured power can also be expressed as:

$$P = \mathbf{g}^{rec} \cdot [\mathbf{W}]\mathbf{g}^{red}$$

Mathematical Characterization Of Scatterers: Stokes Scattering Operator

- From the earlier definition of the Stokes vector, we note that the Stokes vector can be written as:

$$\mathbf{S} = \begin{pmatrix} p_h p_h^* + p_v p_v^* \\ p_h p_h^* - p_v p_v^* \\ p_h p_v^* + p_h^* p_v \\ -i(p_h p_v^* - p_h^* p_v) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -i & i \end{pmatrix} \begin{pmatrix} p_h p_h^* \\ p_v p_v^* \\ p_h p_v^* \\ p_h^* p_v \end{pmatrix} = [\mathbf{R}]\mathbf{g} \quad \Rightarrow \quad \mathbf{g} = [\mathbf{R}]^{-1}\mathbf{S}$$

- This means that we can express the measured power as:

$$P = \mathbf{S}^{rec} \cdot \widetilde{[\mathbf{R}]}^{-1}[\mathbf{W}][\mathbf{R}]^{-1}\mathbf{S}^{rad} \equiv \mathbf{S}^{rec} \cdot [\mathbf{M}]\mathbf{S}^{rad}$$

- The matrix $[\mathbf{M}]$ is known as the *Stokes scattering operator*. It is also called Stokes matrix.
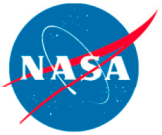
Polarization Synthesis

- Once the scattering matrix, covariance matrix, or the Stokes matrix is known, one can *synthesize* the received power for any transmit and receive antenna polarizations using the polarization synthesis equations:

Scattering matrix: $\qquad P = \left| \mathbf{p}^{rec} \cdot [\mathbf{S}] \mathbf{p}^{rad} \right|^2$

Covariance Matrix: $\qquad P = \mathbf{A} \cdot [\mathbf{C}] \mathbf{A}^*$

Stokes scattering operator: $\qquad P = \mathbf{S}^{rec} \cdot [\mathbf{M}] \mathbf{S}^{rad}$

- Keep in mind that all matrices in the polarization synthesis equations must be expressed in the *backscatter alignment* coordinate system.
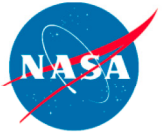
Polarization Synthesis

Polarimetry Demonstration
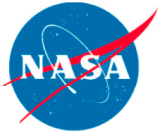
Polarimeter Implementation

- To fully characterize the scatterer, one must measure the full scattering matrix:

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix}^{rec} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} E_h \\ E_v \end{pmatrix}^{tr}$$

- Setting one of the elements of the transmit vector equal to zero allows one to measure two components of the scattering matrix at a time:

$$\begin{pmatrix} S_{hh} \\ S_{vh} \end{pmatrix} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^{inc} \quad ; \quad \begin{pmatrix} S_{hv} \\ S_{vv} \end{pmatrix} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}^{inc}$$

- This technique is commonly used to implement airborne and spaceborne SAR polarimeters.
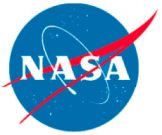
Polarimeter Implementation
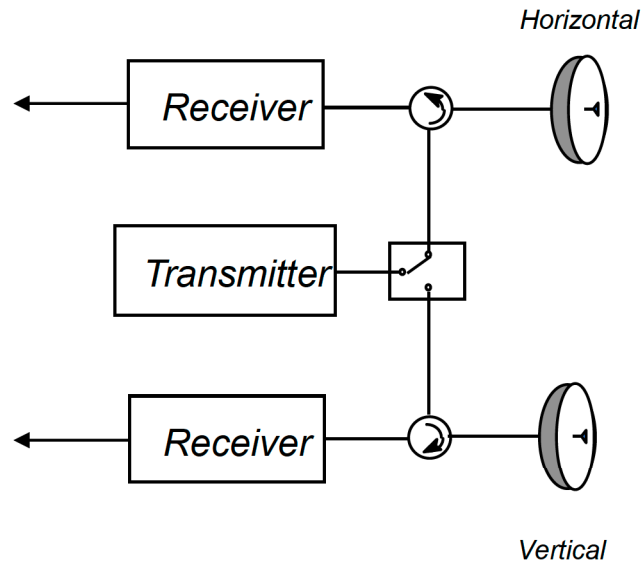
Polarimeter Implementation

- In the implementation shown, the scattering matrix elements are measured in pairs. The two pairs are measured at different times, *i.e.* from (slightly) different viewing directions.

- Due to the speckle effect, this means that the two pairs of signals may not be fully correlated.

- In the airborne case, this decorrelation is of little consequence, since the distance traveled by the aircraft between successive pulses (~ 50 cm) is small compared to the system resolution (~ 2-5 m).

- In spaceborne cases, the decorrelation distance is much closer in length to the distance traveled by the antenna during the interpulse period. Therefore, significant decorrelation could occur. This could be corrected through resampling (interpolation) of one channel with respect to the other.
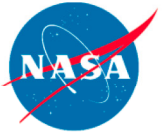
Section 3

POLARIMETRIC DATA ANALYSIS
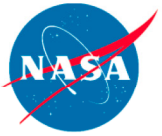
Polarization Signature

- The polarization signature (also known as the polarization response) is a convenient graphical way to display the received power as a function of polarization.

- Usually displayed assuming identical transmit and receive polarizations (*co-polarized*) or orthogonal transmit and receive polarizations (*cross-polarized*).
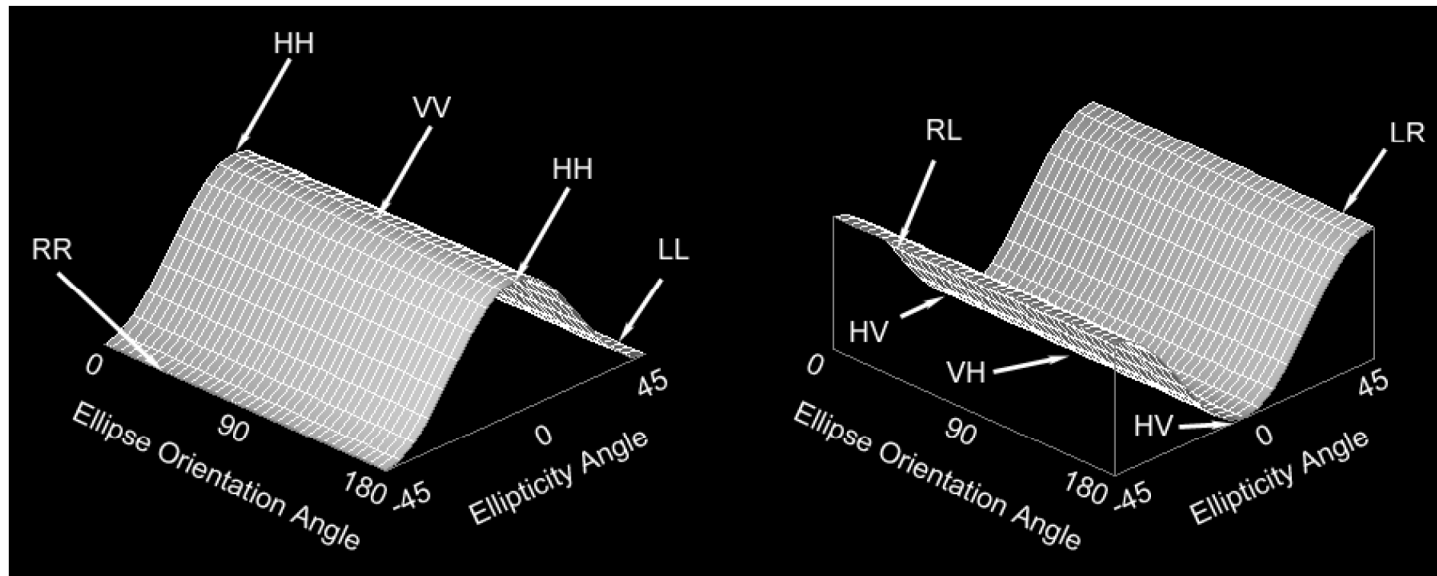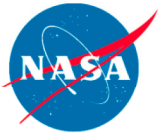
Polarization Signature in 3D

- Another way to look at the signature is that the scatter changes the co-polarized return from a sphere (using the Poincare representation) to some other figure

Co-polarized Response   Cross-polarized Response

Metallic Dihedral Corner Reflector

Corner Reflector Rotated by 45 Degrees

Co-polarized Response

Cross-polarized Response

Vertically Oriented Dipole

Co-polarized Response

Cross-polarized Response

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Dipoles Oriented Cosine^2 about the Vertical

Co-polarized Response

Cross-polarized Response

Dipoles Uniformly Randomly Oriented

Co-polarized Response

Cross-polarized Response

Polarization Signatures
San Francisco, California

Black Forest, Germany

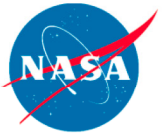# Eigenvalue Decomposition

- Cloude showed that a general covariance matrix $[\mathbf{T}]$ can be decomposed as follows:

$$[\mathbf{T}] = \lambda_1 \mathbf{k}_1 \bullet \mathbf{k}_1^\dagger + \lambda_2 \mathbf{k}_2 \bullet \mathbf{k}_2^\dagger + \lambda_3 \mathbf{k}_3 \bullet \mathbf{k}_3^\dagger + \lambda_4 \mathbf{k}_4 \bullet \mathbf{k}_4^\dagger$$

- Here, $\lambda_i, i = 1,2,3,4$ are the eigenvalues of the covariance matrix, $\mathbf{k}_i, i = 1,2,3,4$ are its eigenvectors, and $\mathbf{k}_i^\dagger$ means the adjoint (complex conjugate transposed) of $\mathbf{k}_i$.

- In the monostatic (backscatter) case, the covariance matrix has one zero eigenvalue, and the decomposition results in at most three nonzero covariance matrices.

- This is similar to a principal component decomposition used in image analysis

- For terrain with reflection symmetry, the eigenvectors can be interpreted to represent odd numbers of reflection, even numbers of reflection and diffuse scattering
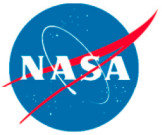
Example of Eigenvalue Decomposition
San Francisco, California

-180   180

HH-VV Phase Difference

|HH-VV| |HV| |HH+VV|
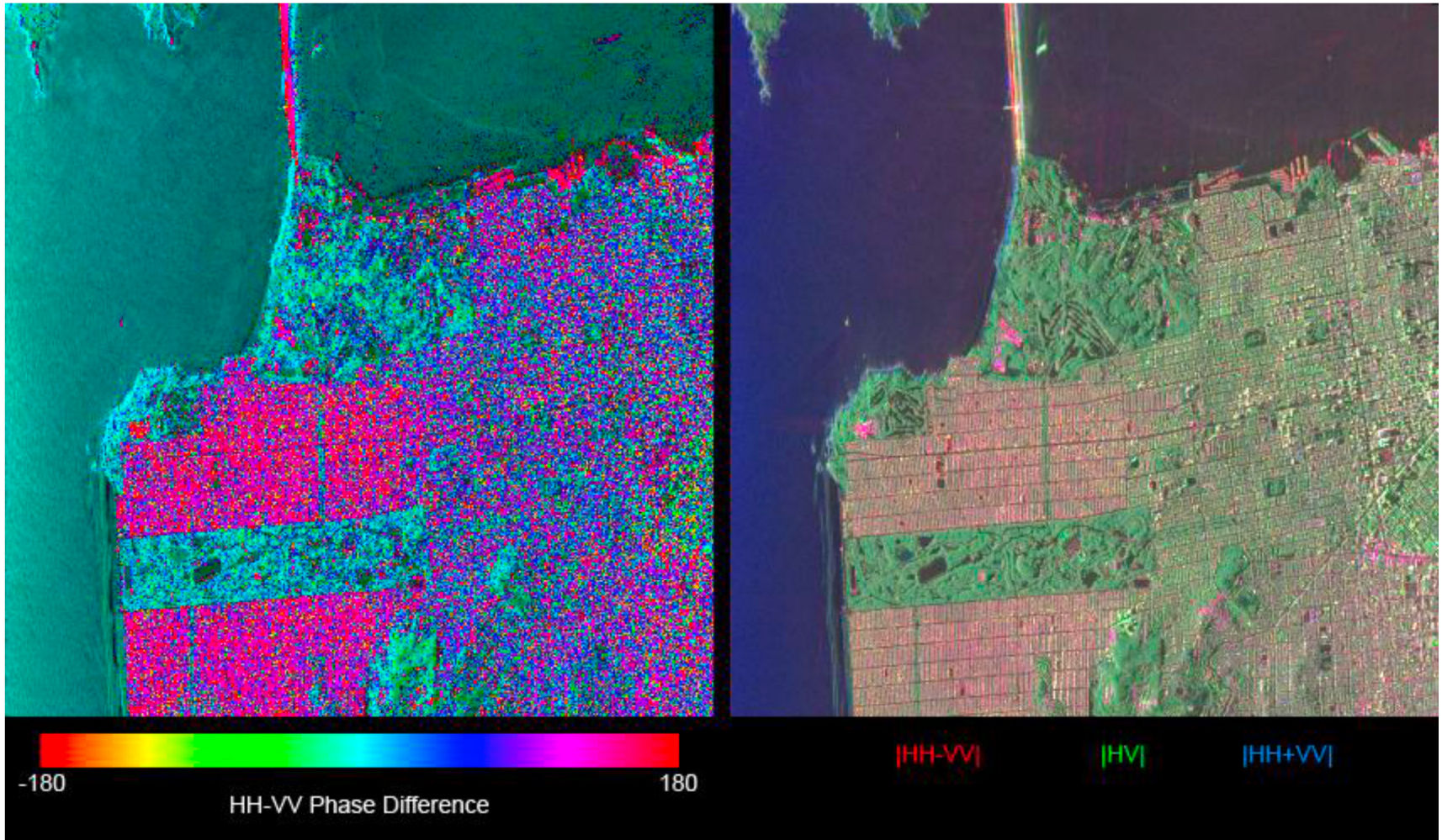
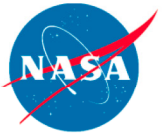# Polarimetric Measures of Randomness

- Polarimetric Entropy

$$H_T = -\sum_{i=1}^{3} P_i \log_3 P_i; \quad P_i = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$$

- Pedestal Height

$$Pedestal\ Height = \frac{\min(\lambda_1, \lambda_2, \lambda_3)}{\max(\lambda_1, \lambda_2, \lambda_3)}$$
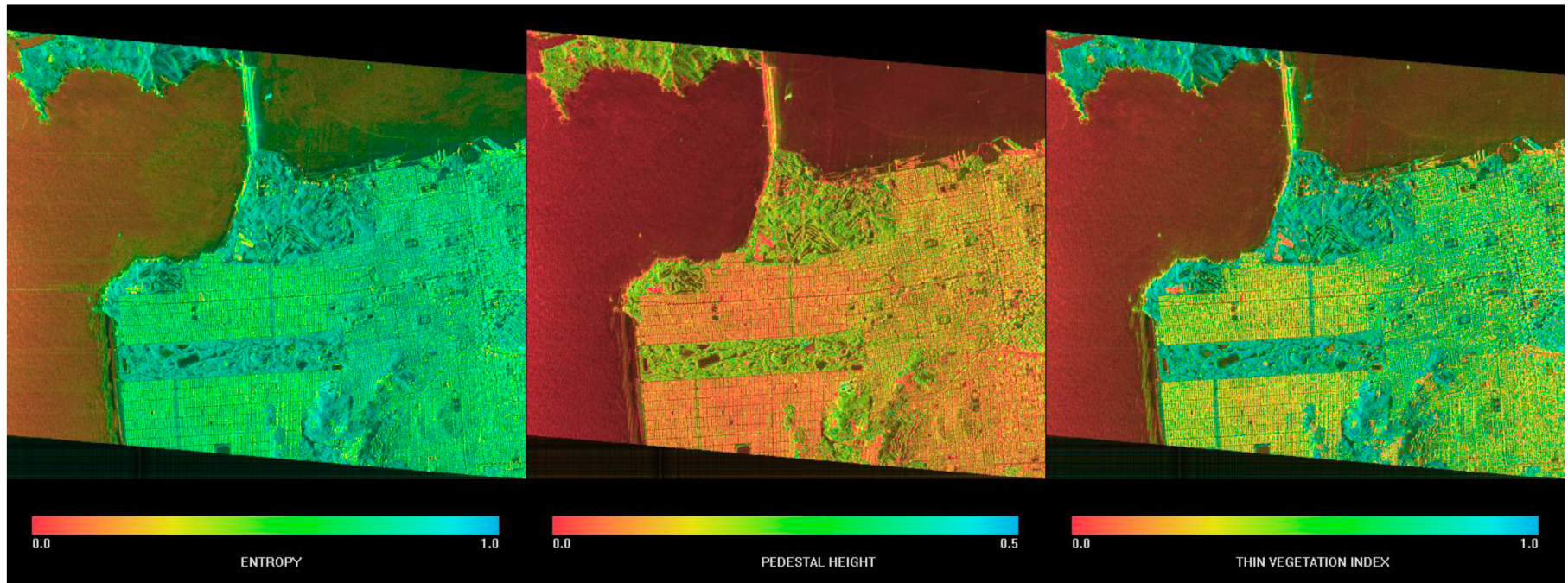
- Radar Vegetation Index

$$RVI = \frac{4\min(\lambda_1, \lambda_2, \lambda_3)}{\lambda_1 + \lambda_2 + \lambda_3} \approx \frac{8\sigma_{hv}}{\sigma_{hh} + \sigma_{vv} + 2\sigma_{hv}}$$
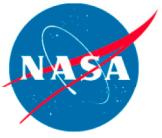
Polarimetric Randomness
San Francisco, California

ENTROPY — 0.0 to 1.0

PEDESTAL HEIGHT — 0.0 to 0.5

THIN VEGETATION INDEX — 0.0 to 1.0

Multi-Frequency Radar Vegetation Index
Black Forest, Germany

C-Band   L-Band   P-Band

The Alpha Angle and Anisotropy

- Cloude and Pottier proposed the following description for each of the eigenvectors of the coherency matrix

$$\tilde{\mathbf{e}} = \left(\cos\alpha \quad \sin\alpha\cos\beta e^{i\delta} \quad \sin\alpha\sin\beta e^{i\gamma} \right)$$

- The average alpha angle is calculated as

$$\bar{\alpha} = \sum_{i=1}^{3} \alpha_i P_i$$

- The average alpha angle varies from zero for trihedral corner reflectors to 90 degrees for dihedral scattering. Dipole scattering represents an intermediate case with alpha = 45 degrees.
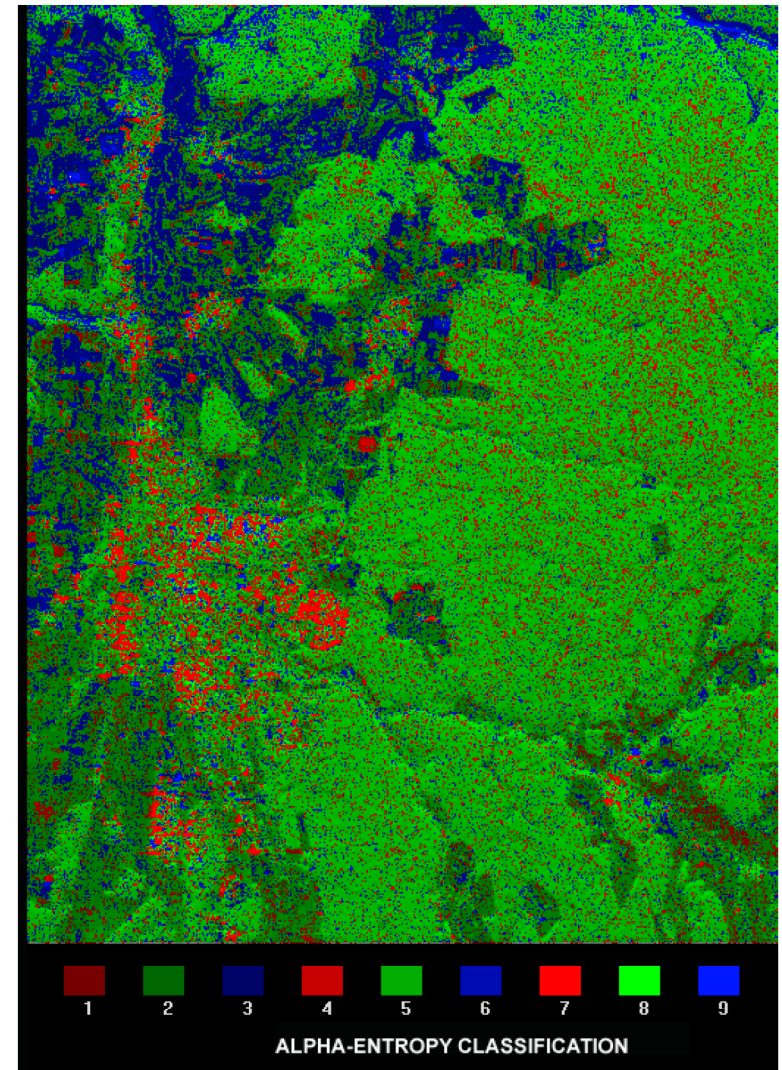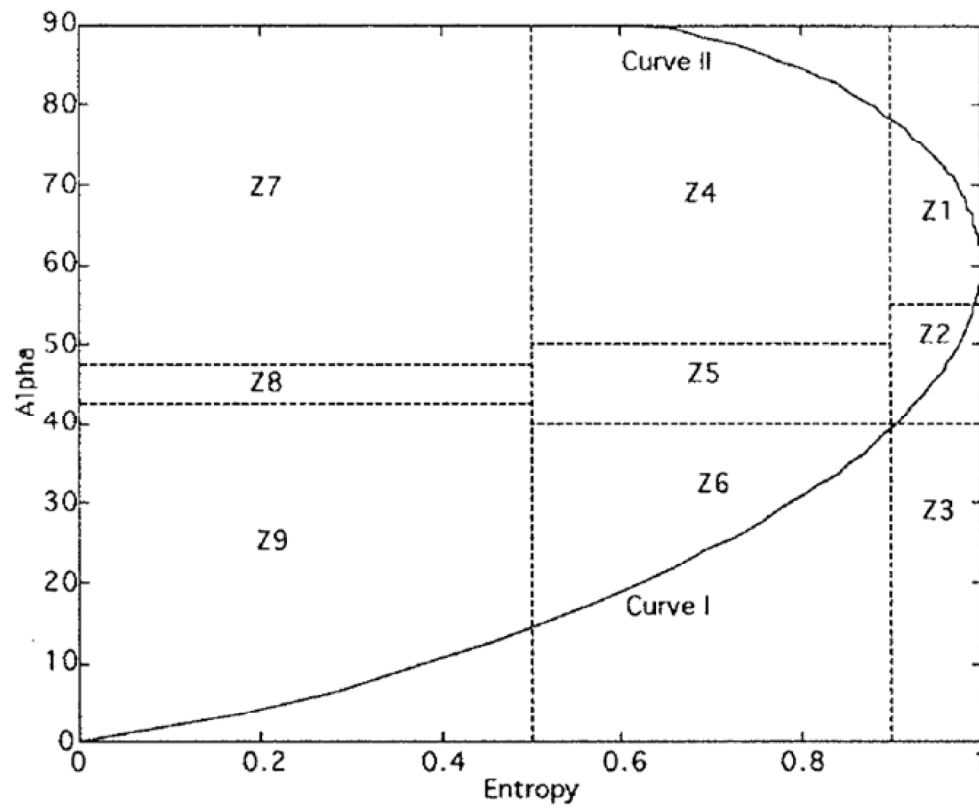
- The anisotropy is defined as

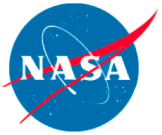$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$
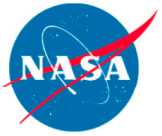
Alpha/Entropy Classification

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Model Based Decomposition of Scattering

- Model based decompositions attempt to interpret the observed scattering based on known models

- The observed covariance matrix is written as

$$\langle [\mathbf{C}] \rangle = a[\mathbf{C}_{model1}] + b[\mathbf{C}_{model2}] + c[\mathbf{C}_{model3}] + [\mathbf{C}_{remainder}]$$

- The allowable values of the coefficients *a, b* and *c* are calculated from the requirement that each of the matrices have to have positive eigenvalues

- Allowable values typically occupy a volume in space

- If at each step we allow the maximum value of the model contribution, the allowable values fall an a surface

- One such scheme is the three component decomposition published by Freeman and Durden
    - Unfortunately their scheme often leads to negative eigenvalues
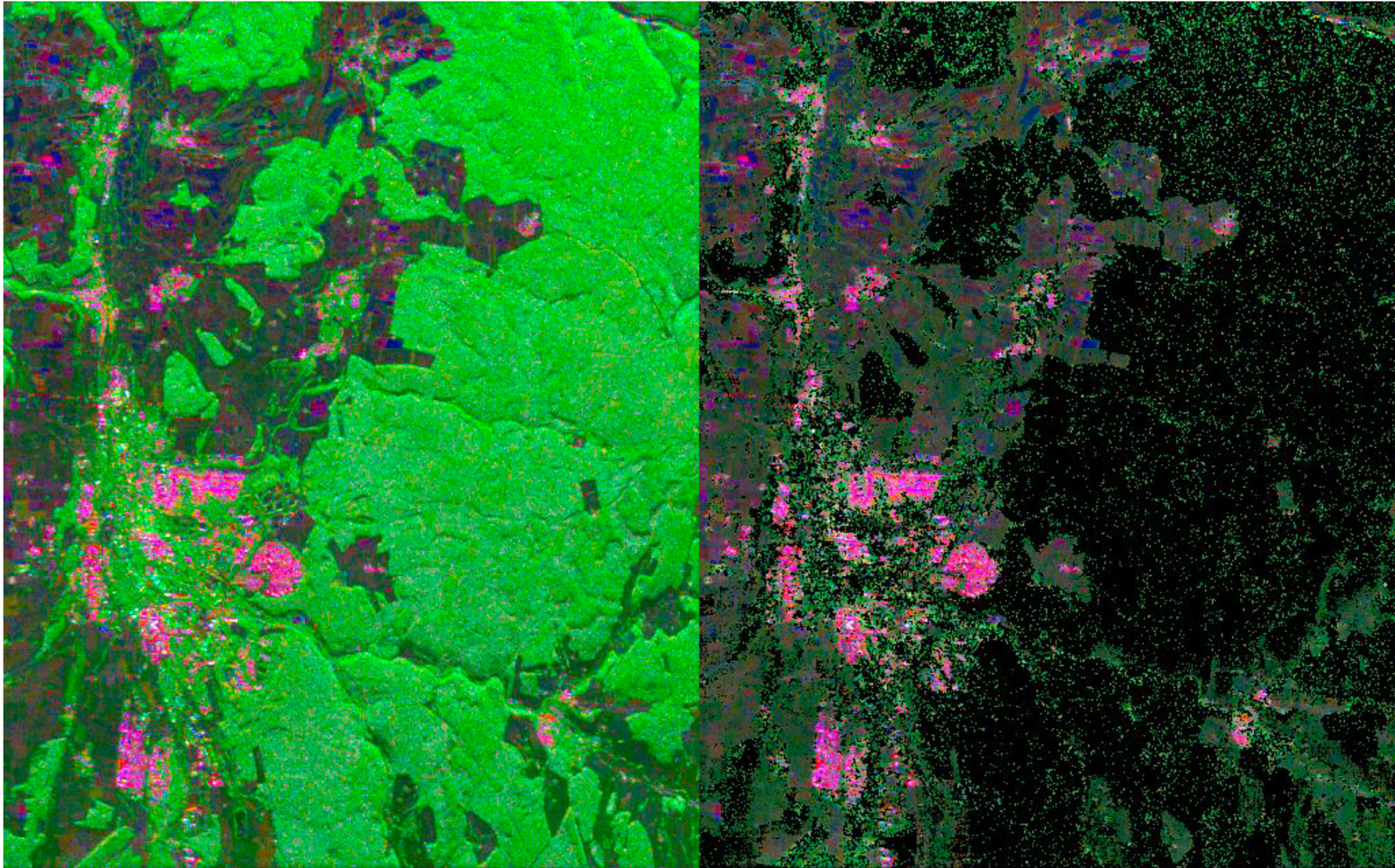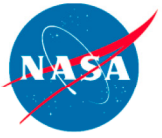   - Can be fixed by appropriate scaling of the random vegetation contribution

Model Based Decomposition: Freeman and Durden Negative Eigenvalues are Masked in Black

JvZ-63

Model Based Decomposition
Added Constraint of Positive Eigenvalues

Freeman and Durden Decomposition

Modified Three Component Decomposition

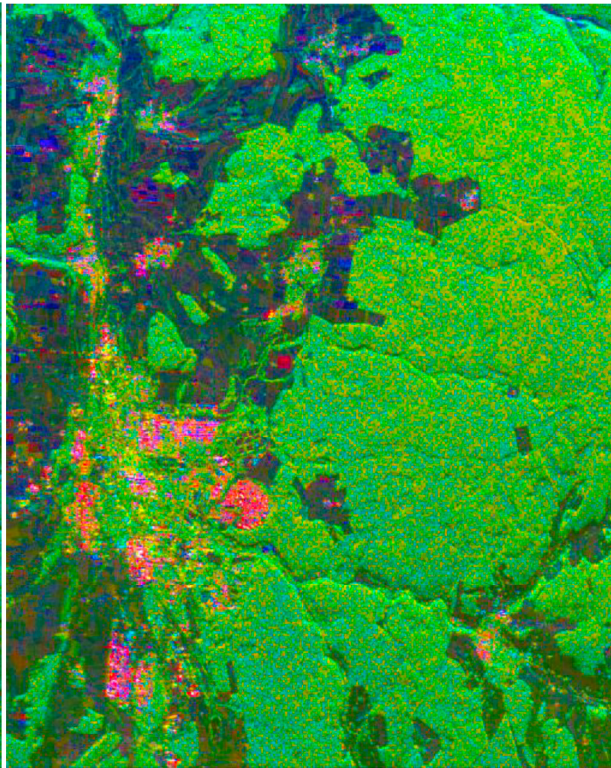JvZ-64

Multi-Frequency Decomposition:
Black Forest, Germany

C-Band

L-Band

P-Band

Polarimetry Books for Further Reading